Module 2: Real Formulas

On each of the following slides you will be given a formula or equation and information about what to do with it.

You will be graded on showing all of your work with correct explanations and steps.

You may use a Jamboard, Sketchtoy, or paper/pencil to hand-write your work and upload photos into your slides. ***IF*** you choose to type, you will need to use correct mathematical type (**EX**: x^2 not x^2)

#1: Distance

The distance traveled by an object can be modeled by the equation

$$d = ut + 0.5at^2$$

d = distance
u = initial velocity
t = time
a = acceleration.

a. Show all work to solve the formula for ***a***.
b. What is the acceleration of someone who traveled 94 miles with an initial velocity of 30 mph and drove for 2 hours? Show all work.

Show work here

To solve for a:

A] Subtract "ut" from both sides

$$d = ut + 0.5at^2$$

$$-ut = -ut$$

$$d - ut = 0.5at^2$$

B] To clear decimal, multiply both sides by 2

$$2(d - ut) = (2)(0.5)at^2$$

$$2(d - ut) = (1)\,at^2$$

$$\frac{2(d-ut)}{t^2} = a$$

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Show work here

$$\frac{2(d-ut)}{t^2} = a, \quad \text{for } d = 94,\ u = 30,\ t = 2$$

$$\frac{2[94-(30)(2)]}{2^2} = a$$

$$\frac{2[94-(60)]}{2^2} = a$$

$$\frac{2(34)}{2^2} = \frac{2((2)(17))}{2^2} = a = 17$$

$$a = 17$$

The acceleration is 17 mi/hr per hour

Check
(94 mi) =? $(30mi/hr)(2hr) + 0.5(17mi/hr^2)(2hr)^2$
94 mi =? 60 mi + 34 mi
94 mi = 94 mi CORRECT.

#2: Distance

The distance traveled by a falling object can be modeled by the equation

$$d = 0.5gt^2$$

d = distance
t = time
g = acceleration due to gravity.

a. Show all work to solve the formula for ***g***.
b. Show all work to find the acceleration due to gravity if it takes a baseball 10 seconds to hit the ground after being dropped from a height of 490 meters.

Show work here

$$d = 0.5gt^2$$

Solve for g.

To clear decimals, multiply

both sides of the equation by 2

$$(2)d = (2)0.5gt^2$$

$$2d = (1)gt^2$$

Divide both sides by t^2

$$g = \frac{2d}{t^2}$$

#2: Distance

The distance traveled by a falling object can be modeled by the equation

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d = distance
t = time
g = acceleration due to gravity.

a. Show all work to solve the formula for ***g***.
b. Show all work to find the acceleration due to gravity if it takes a baseball 10 seconds to hit the ground after being dropped from a height of 490 meters.

$$g = \frac{2d}{t^2},$$

where d = 490 m, t =10 sec

$$g = \frac{2(490)}{10^2} = 9.80\ m/s^2$$

This agrees with the acceleration of gravity, which is 9.8 m/s per second.

#3: SeeSaw

Two boys want to use a seesaw but they need to adjust it so the weight balances out. The formula for this is

$$w_1 d_1 = w_2 d_2$$

w_1= weight of the first boy 165 lb
w_2 = weight of second boy 110 lb
d_1 = distance of first boy from the fulcrum 4 in
d_2 = distance of second boy from fulcrum 6 in

a. Show all work to solve this formula for d_2.
b. Then show all work to find out how far the second boy needs to be from the fulcrum if he weighs 110 lbs, the first boy weighs 165 lbs and is 4 inches away from the fulcrum.

CHECK: $w_1 d_1 = w_2 d_2$
(165 lb)(4 in) = (110 lb)(6 in)
660 lb-in = 660 lb-in

Show work here

$$w_1 d_1 = w_2 d_2$$

Divide both sides by d_2

$$d_2 = \frac{w_1 d_1}{w_2}$$

When $w_2 = 110\ lb$,

$w_1 = 165\ lb$, $d_1 = 4\ in$

$$d_2 = \frac{(165\ lb)(4\ in)}{(110\ lb)} = 6\ in$$

#4: Fence

Ellen wants to put grass in and fence part of her yard, which is in the shape of a triangle attached to a rectangle. (see photo) The area of this yard can be found by using the formula

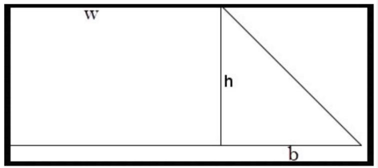
$A = (wh) + \frac{1}{2}(bh)$

A = area of the yard

w = width of rectangular

h = height of the rectang

b = base of the triangula



a. Show all work to solve the formula for b.

b. Show all work to find how much fencing she would need for the base of the triangular part of the yard if the width is 100 feet and the height is 80 feet and the area of the entire yard is 10,400 sq ft.

Show work here

$A = (wh) + \frac{1}{2}(bh)$

Clear fractions, multiply 2X both sides

$2A = 2wh + bh$

$-2wh = -2wh$ Subtract 2wh

$2A - 2wh = bh$ Divide by h both sides

$\frac{2A - 2wh}{h} = b$

For $A = 10{,}400\ ft^2,\ w = 100\ ft,\ h = 80\ ft$

$B = \frac{2A - 2wh}{h} = \frac{2(10{,}400) - 2(100)(80)}{80} = \frac{20{,}800 - 16{,}000}{80}$

$B = \frac{4800}{80} = 60$ ft

CHECK: **10,400 ft^2** = (100 ft)(80ft) + (0.5)(80 ft)(60ft)

= 8000 ft^2 + (2400ft^2) = **10,400 ft^2**

#5: You Choose!

On the next 2 slides are algebraic formulas that are used in real situations. You get to choose which formula you will manipulate, and which variable in that formula you will solve for!

Your answer should look like this:

- I chose the area formula, $A = (\ell)(w)$ and I am solving for the variable w.

$$\frac{A}{\ell} = \frac{(\ell)(w)}{\ell}$$

$$A / \ell = w$$

Show work here

FORMULA CHOICES

Your parents/guardians told you that for every ***y*** minutes you read, you earn ***x*** minutes of video game time.

$$\frac{\frac{x}{20} \cdot 30}{60} = y$$

x = number of minutes you want to play
y = number of hours you get to play.

a. Show all work to solve for *x.*
b. Show all work to see how much you would get to play if you read for 180 minutes.

The formula to find your BMI (body mass index) is

$$B = \frac{m}{h^2}$$

B = body mass index
m = mass (in kilograms)
h = height (in meters)

a. Show all work to solve for *h.*
b. Find the height of a woman who weighs 68 kg and has a BMI of 21.

FORMULA CHOICES

The formula to find your BMI (body mass index) is

$$B = \frac{m}{h^2}$$

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a. Show all work to solve for h.
b. Find the height of a woman who weighs 68 kg and has a BMI of 21.

FORMULA CHOICES

$B = \frac{m}{h^2}$

$B\,(h^2) = \frac{m}{h^2}(h^2)$ *(Multiply both sides by* (h^2)*)*

$B\,(h^2) = m$ *(*h^2 *cancels on right)*

$(h^2) = \frac{m}{B}$ *(Divide both by B)*

$h = \sqrt{\frac{m}{B}}$ *(Take square root to isolate h)*

$B = \frac{m}{h^2}$

FORMULA CHOICES

Find the height of a woman who weighs 68 kg and has a BMI of 21.

$$h = \sqrt{\frac{m}{B}} = \sqrt{\frac{68}{21}} = h = \sqrt{3.2381} = 1.7885\ m$$

FORMULA CHOICES

A cell phone company charges $40 a month for 300 minutes, Extra calls above the 300 minutes cost $0.50 per minute.

They use the formula to calculate the total monthly cost, in dollars.

$\boldsymbol{T} = 40 + (\boldsymbol{m} - 300)(0.50)$

T = total cost of the phone plan
m = minutes used in a month

a. Show all your work to solve for *m*.
b. How many minutes did you use if your bill was $380.50? Show all work.

The formula to convert temperature from Celsius to Fahrenheit is given by

$$F = C\left(\frac{9}{5}\right) + 32$$

Where *F* is Fahrenheit and *C* is Celsius.

a. Show all work to solve for *C*.
b. Find the temperature in Celsius if it is 104^{o} fahrenheit. Show all work.

FORMULA CHOICES

The formula to convert temperature from Celsius to Fahrenheit is given by

$$F = C\left(\frac{9}{5}\right) + 32$$

Where *F* is Fahrenheit and *C* is Celsius.

a. Show all work to solve for *C*.
b. Find the temperature in Celsius if it is 104° fahrenheit. Show all work.

$$F = C\left(\tfrac{9}{5}\right) + 32$$

$$-32 = \quad -32 \qquad \text{(Subtract 32 both sides)}$$

$$F - 32 = C\left(\tfrac{9}{5}\right)$$

$$\left(\tfrac{5}{9}\right)(F - 32) = C\left(\tfrac{9}{5}\right)\left(\tfrac{5}{9}\right) \qquad \text{(Multiply both sides by } \left(\tfrac{5}{9}\right)\text{)}$$

$$\left(\tfrac{5}{9}\right)(F - 32) = C \qquad \text{(Reorder)}$$

$$C = \left(\tfrac{5}{9}\right)(F - 32)$$

FORMULA CHOICES

The formula to convert temperature from Celsius to Fahrenheit is given by

$$F = C\left(\frac{9}{5}\right) + 32$$

Where *F* is Fahrenheit and *C* is Celsius.

a. Show all work to solve for *C*.
b. Find the temperature in Celsius if it is 104° fahrenheit. Show all work.

For 104F

$$C = \left(\frac{5}{9}\right)(F - 32)$$

$$C = \left(\frac{5}{9}\right)(104 - 32)$$

$$C = \left(\frac{5}{9}\right)(72) = (5)\left(\frac{72}{9}\right)$$

$$C = (5)(8) = 40$$

$$104\ F = 40\ C$$

HONORS ONLY

The quadratic formula is a formula used to solve for ***x*** values in a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Show all work to solve it for ***c***.

Here is a brief video to help.

POSITIVE SOLUTION

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Positive Solution

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

(Multiply both sides by 2a)

$$2ax = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}(2a)$$

(Add b to both sides)

$$2ax = -b + \sqrt{b^2 - 4ac}$$

$$+b = +b + \sqrt{b^2 - 4ac}$$

$$2ax + b = + \sqrt{b^2 - 4ac}$$

(Square both sides)

$$(2ax + b)^2 = b^2 - 4ac$$

$$(2ax + b)(2ax + b) = b^2 - 4ac$$

$$(2ax + b)(2ax + b) = b^2 - 4ac$$

(Distribute multiplication)

$$(2ax + b)(2ax) + (2ax + b)(b) = b^2 - 4ac$$

$$4a^2x^2 + (2abx + 2abx) + b^2 = b^2 - 4ac$$

$$- b^2 = - b^2$$

(Combine like terms and subtract b^2 from both sides)

$$4a^2x^2 + 4abx = - 4ac$$

(Divide both sides by -4a)

$$c = - \frac{4a^2x^2 + 4abx}{4a} = - \frac{ax^2 + bx}{1} * \frac{4a}{4a}$$

$$c = - (ax^2 + bx)$$

$$-c = ax^2 + bx$$

$$ax^2 + bx + c = 0$$

NEGATIVE SOLUTION

HONORS ***ONLY***

The quadratic formula is a formula used to solve for ***x*** values in a quadratic equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Show all work to solve it for ***c***.

Here is a brief video to help.

Negative Solution

$$x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

(Multiply both sides by 2a)

$$2ax = \frac{-b - \sqrt{b^2 - 4ac}}{2a}(2a)$$

(Add b to both sides)

$$2ax = -b - \sqrt{b^2 - 4ac}$$

$$+b = +b - \sqrt{b^2 - 4ac}$$

$$2ax + b = - \sqrt{b^2 - 4ac}$$

$$2ax + b = - \sqrt{b^2 - 4ac}$$

$$(-1)(2ax \quad +b) = (-1)(-\sqrt{b^2 - 4ac})$$

$$(-1)(2ax \quad +b) = \sqrt{b^2 - 4ac})$$

(Multiply both sides by negative 1 to clear negative in front of square root)
(Square both sides)

$$(-1)(2ax \quad +b)\,(-1)(2ax \quad +b) = (\sqrt{b^2 - 4ac})(\sqrt{b^2 - 4ac})$$

(On the left side, (- 1)(-1) = 1. We are back to the same math as the positive solution now.)

$$(2ax + b)^2 = b^2 - 4ac$$

$$(-1)(2ax + b)(-1)(2ax + b) = (\sqrt{b^2 - 4ac})(\sqrt{b^2 - 4ac})$$

(On the left side, (- 1)(-1) = 1. We are back to the same math as the positive solution now!)

$$(2ax + b)^2 = b^2 - 4ac$$

$$(2ax + b)(2ax + b) = b^2 - 4ac$$

(Distribute multiplication)

$$(2ax + b)(2ax) + (2ax + b)(b) = b^2 - 4ac$$

$$4a^2x^2 + (2abx + 2abx) + b^2 = b^2 - 4ac$$

$$-b^2 = -b^2$$

(Combine like terms and subtract b^2 from both sides)

$$4a^2x^2 + 4abx = -4ac$$

$$4a^2x^2 + 4abx = - 4ac$$

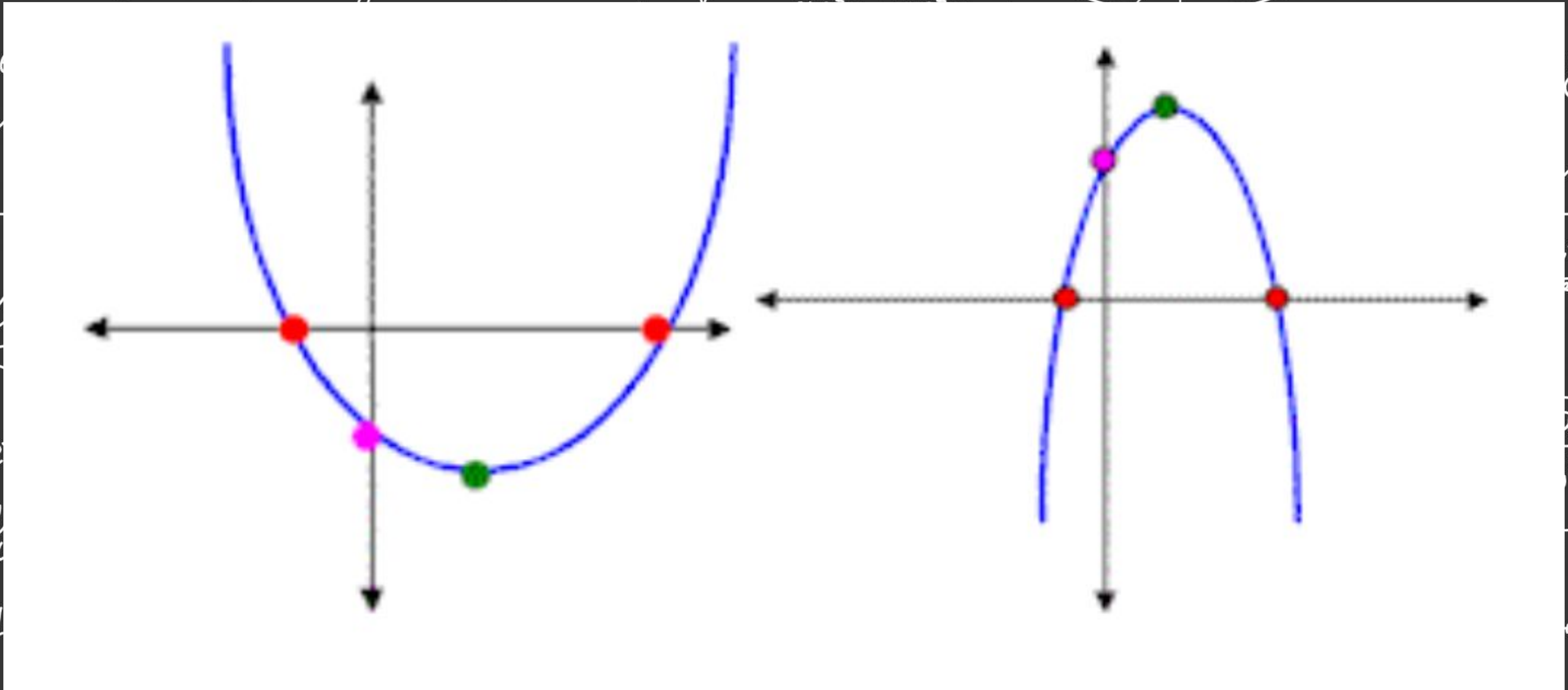
(Divide both sides by -4a)

$$c = -\frac{4a^2x^2 + 4abx}{4a} = -\frac{ax^2 + bx}{1} * \frac{4a}{4a}$$

$$c = -(ax^2 + bx)$$

$$-c = ax^2 + bx$$

$$ax^2 + bx + c = 0$$



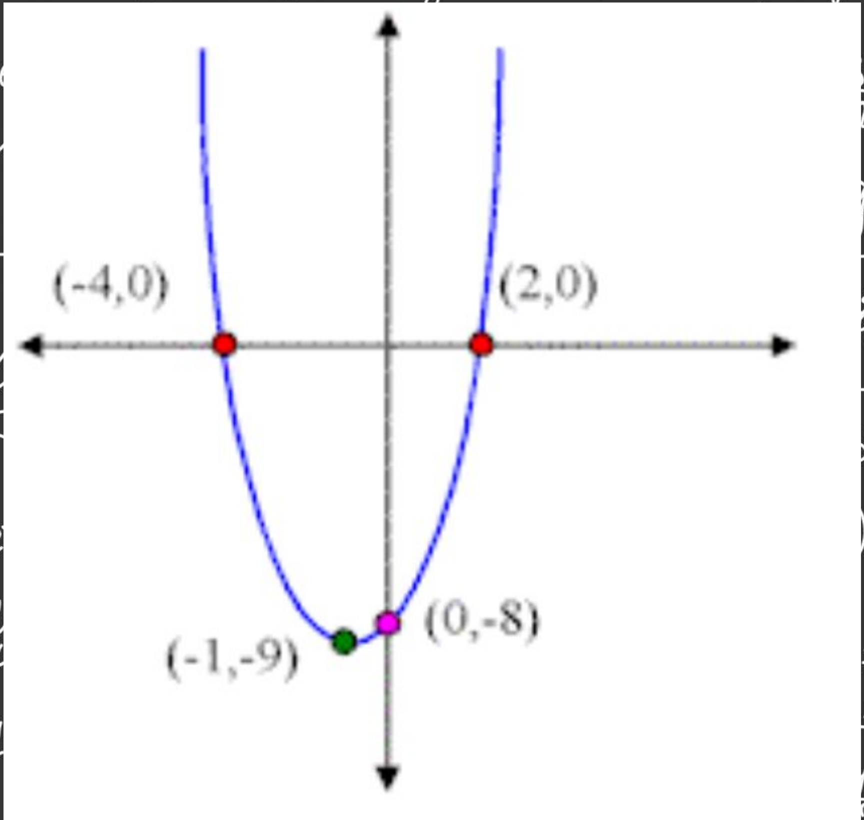
$y = ax^2 + bx + c$

Pink dot: x = 0, y-intercept, at (0,c)

PARABOLA

$$y = ax^2 + bx + c$$

$$y = x^2 + 2x - 8$$



$y = (x + 4)(x - 2)$, Root $y = 0$ at $x = -4$, $x = +2$

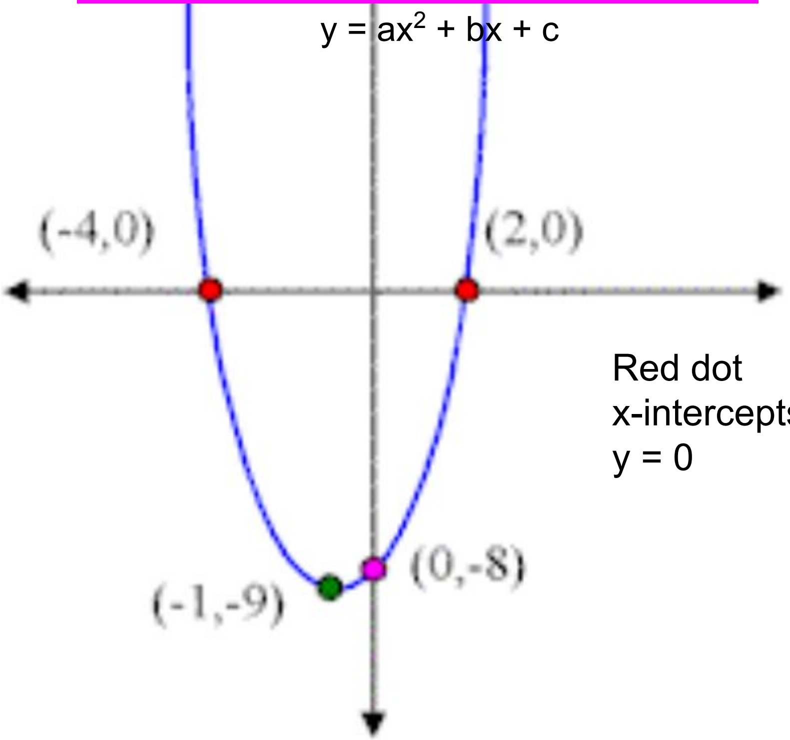
Pink dot: $x = 0$,
y-intercept, at $(0,c)$

To find the vertex we use: $h = \frac{-b}{2a}$

If $a > 0$ (positive) then the parabola opens upward.
If $a < 0$ (negative) then the parabola opens downward.

PARABOLA $y = x^2 + 2x - 8$

$y = ax^2 + bx + c$



Red dot
x-intercepts,
y = 0

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

a = 1, b = 2, c = -8

$$x = \frac{-2 - \sqrt{2^2 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{-2 - \sqrt{4+32)}}{2(1)}$$

$$x = \frac{-2 - \sqrt{36)}}{2(1)}$$

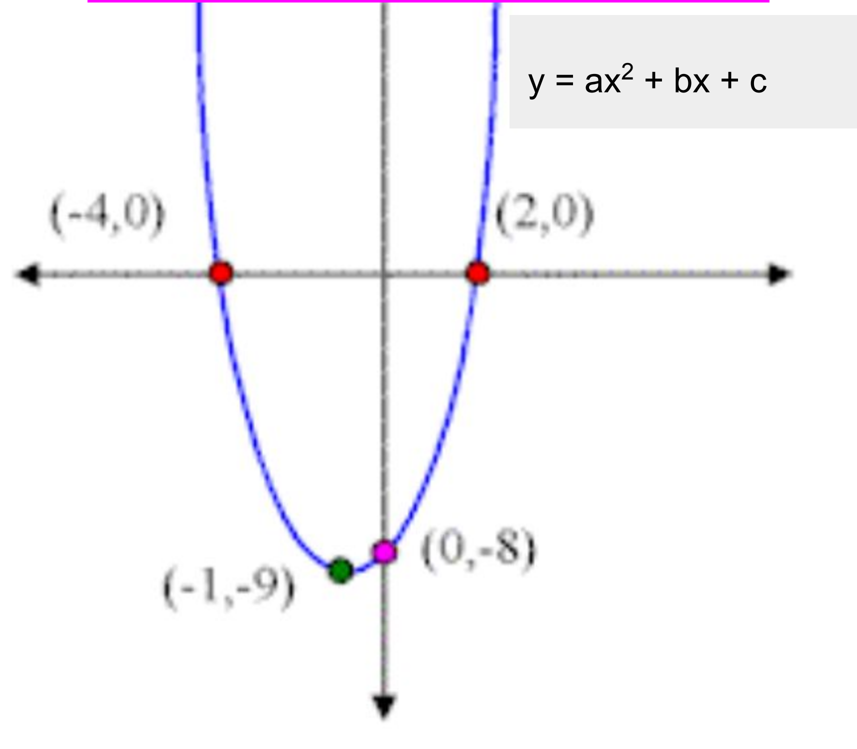
$$x = \frac{-2 - 6}{2(1)} = \frac{-8}{2}$$

$$x = -4$$

Negative solution

PARABOLA $y = x^2 + 2x - 8$

$y = ax^2 + bx + c$



$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1, \; b = 2, \; c = -8$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Positive Solution

$$x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$a = 1, b = 2, c = -8$

$$x = \frac{-2 + \sqrt{2^2 - 4(1)(-8)}}{2(1)} = \frac{-2 + \sqrt{4 + 32)}}{2(1)}$$

$$x = \frac{-2 + \sqrt{36}}{2(1)} = \frac{-2 + 6}{2(1)} = \frac{4}{2}$$

$$x = 2$$

$Y = x^2 + 2x - 8 = (x - 2)\,(x + 4)$

PARABOLA $y = x^2 + 2x - 8$

$a = 1, \ b = 2, \ c = -8$

$y = ax^2 + bx + c$